LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2007

MT 1804 - LINEAR ALGEBRA

AB 19

Date : 25/10/2007 Time : 1:00 - 4:00 Inswer ALL Questions. a) i) Let T be the linear operator on R^3 which is represented in the standard of basis by the matrix $\begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$. Prove that T is diagonalizable by exit basis of R^3 , each vector of which is a characteristic vector of T. Or ii) Let T be a linear operator on a finite dimensional vector space V. Let $c_1, c_2 \dots c_k$ be the distinct characteristic values of T and let W_i be the null $(T - c_i D)$. If the characteristic polynomial for T is $f = (x - c_1)^{d_1} (x - c_2)^{d_2} \dots$ dim $W_i = d_i$, i=1, 2 k then prove that dim $V = \dim W_i + \dim W_2 + \dots + di$ b) i) Let T be a linear operator on a finite dimensional vector space V. Prove minimal polynomial for T divides the characteristic polynomial for T. Or ii) Let V be a finite dimensional vector space V. Prove minimal polynomial for T divides the characteristic polynomial for T. Or ii) Let V be a finite dimensional vector space ver F and T be a linear oper then prove that T is diagonalizable if and only if the minimal polynomi the form $p = (x - c_1)(x - c_2) \dots (x - c_k)$ where $c_1, c_2 \dots c_k$ are distinct elen II. a) i) Define independent subspaces of a vector space V. ii) Let V be a finite dimensional vector space V. iii) Let V be a finite dimensional vector space V. iii) Let V be a linear operator on a finite dimensional vector space V. iii) Let V be a linear operator on a finite dimensional vector space V. iii) Let V be a linear operator on a finite dimensional vector space V and let $E_1, E_2, \dots E_k$ are linear operators on V such that 1) each E_i is a projectior 2) $E_i E_j = 0$ if $i \neq j$ 3) $I = E_1 + E_2 + \dots E_k$ and let W_i is the range of E_i . I W_i is invariant under T then prove that T $E_i = E_i$ T, i=1,2 k b) i) State and prove primary decomposition theorem.	hibiting a space of $(x-c_k)^{d_k}$, im W_k (5) that the rator on V ial for T has hents of F.	Marks (15)
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$W_{j} \cap (W_{1} + W_{2} + + W_{j-1}) = \{0\}$ Or iii) Let T be a linear operator on a finite dimensional vector space V and let E_{1}, E_{2}, E_{k} are linear operators on V such that 1) each E_{i} is a projection 2) $E_{i}E_{j} = 0$ if $i \neq j$ 3) $I = E_{1} + E_{2} + E_{k}$ and let W_{i} is the range of E_{i} . If W_{i} is invariant under T then prove that T $E_{i} = E_{i}$ T, $i=1,2$ k b) i) State and prove primary decomposition theorem. Or	$\angle \geq I \geq K$,	
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$E_1, E_2,, E_k$ are linear operators on V such that 1) each E_i is a projection 2) $E_i E_j = 0$ if $i \neq j$ 3) $I = E_1 + E_2 +, E_k$ and let W_i is the range of E_i . I W_i is invariant under T then prove that $T E_i = E_i T$, $i=1,2$ k b) i) State and prove primary decomposition theorem. Or	``	
W_i is invariant under T then prove that T $E_i = E_i$ T, i=1,2 k b) i) State and prove primary decomposition theorem. Or		
b) i) State and prove primary decomposition theorem. Or	f each	
Or	(5)	
ii) Let α be any non-zero vector in V and p_{α} be the T-annihilator of α . T	hen	
prove that if deg $p_{\alpha} = k$ and if U is the linear operator on Z(α ;T) induc		
then k=dim Z(α ;T) and the minimal polynomial for U is p_{α} .	(15)	
$\mathbf{F} = \mathbf{F} \mathbf{a}$	()	
(. a) i) If T is a linear operator on a finite dimensional vector space V, then pro every T-admissible subspace has a complementary subspace which is a invariant under T.		
Or		
$\begin{pmatrix} 2 & 0 & 0 \\ & & & \end{pmatrix}$		
ii) Let A= $\begin{pmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{pmatrix}$ be the complex 3X3 matrix. Prove that A is sire	nilar to	
diagonal matrix if and only if a=0.		

ii) Let T be a linear operator on a finite dimensional vector space V and let W_0 be a proper T-admissible subspace of V. Then prove that there exist non-zero vectors $\alpha_1, \alpha_2, \dots, \alpha_r$ in V with respective T-annihilators p_1, p_2, \dots, p_r such that $V = W_0 \oplus Z(\alpha_1; T) \oplus Z(\alpha_2; T) \dots \oplus Z(\alpha_r; T) \text{ and } p_k \text{ divides } p_{k-1}, k=2,3 \dots r. (15)$ IV. a) i) Let V be a finite dimensional inner product space and f a form on V. Then prove that there is a unique operator T on V such that $f(\alpha, \beta) = (T\alpha / \beta)$ for all α, β in V where (α / β) is the inner product defined on V. Or ii) Let V be a complex vector space and f a form on V. Then prove that f is hermitian if and only if $f(\alpha, \alpha)$ is real for every α . (5) and let g be the form defined by $g(X,Y)=Y^*AX$. Is g an inner b) i) Let A product? ii) State and prove Principle Axis theorem. (6+9)Or iii) Let f be a form on a real or complex vector space V and $\{\alpha_1, \alpha_2, ..., \alpha_r\}$ a basis for the finite dimensional subspace W of V. Let M be the rxr matrix with entries $M_{jk} = f(\alpha_k, \alpha_j)$ and W' the set of all vectors β in V such that f (α, β)=0 for all $\alpha \in W$. Then prove that W' is a subspace of V and $W \cap W' = \{0\}$ if and only if M is invertible and when this is the case V=W+W'. (15)V. a) i) Let V be a vector space over the field F. Define a bilinear form f on V and prove that the function defined by $f(\alpha, \beta) = L_1(\alpha) L_2(\alpha)$ is bilinear. Or ii) State and prove polarization identity for symmetric bilinear form f. (5)b) i) Let V be a finite dimensional vector space over the field of complex numbers. Let f be a symmetric bilinear form on V which has rank r. Then prove that there is an ordered basis $B = \{\beta_1, \beta_2, ..., \beta_n\}$ for V such that the matrix of f in the ordered basis B is diagonal and f $(\beta_i, \beta_j) = \begin{cases} 1, & j=1,2,..r \\ 0, & j>r \end{cases}$ ii) If f is a non-zero skew-symmetric bilinear form on a finite dimensional vector space V then prove that there exist a finite sequence of pairs of vectors, $(\alpha_1, \beta_1), (\alpha_2, \beta_2), ..., (\alpha_k, \beta_k)$ with the following properties: 1) f (α_j, β_j)=1, j=1,2,...,k. 2) f $(\alpha_i, \alpha_j) = f(\beta_i, \beta_j) = f(\alpha_i, \beta_j) = 0, i \neq j.$ 3) If W_i is the two dimensional subspace spanned by α_i and β_i , then $V = W_1 \oplus W_2 \oplus ... W_k \oplus W_0$ where W_0 is orthogonal to all α_i and β_i and the restriction of f to W_0 is the zero form. (15)
